Mass nouns and plural logic

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1. Introduction

Background: in languages like English and French, common nouns are divided in two morphosyntactic classes:

* Mass nouns: *water*, *gold*, *silverware*, *furniture*…
Normally invariable in grammatical number, most being singular; used with determiners like *some*, *a little*, *a lot of*.

* Count nouns: *cat*, *grain*, *pile*…
Used in the singular and in the plural, with determiners like *one*, *every*, *some*, *two*.

Singular terms: terms that refer to a single thing.
Plural term: terms that refer to one or several things.

John fed this cat.	Singular terms
John and Bill fed these cats.	Plural terms
This wine costs a hundred euros.	Seems able to refer to one or several things

NB: The grammatical number of mass nouns is invariable. Therefore, it has no semantic value with these nouns, and does not indicate whether they refer to one or several things.

Existing approaches of mass nouns in terms of sets or mereological sums face problems, and treat mass nouns as singular terms.
Using recent work in a plural logic, can we formulate a better semantics of mass nouns, in which they can refer to several things at once?

Plan

1. Introduction
2. Problems for approaches in terms of sets
3. Problems for approaches in terms of mereological sums
4. Plural logic
5. A new semantics for mass nouns
6. Conclusion

2. Problems for approaches in terms of sets

Strawson (1959), Laycock (1972).

Mass nouns are treated as predicates:
This is wine is true iff I(*this*) ∈ I(*wine*)
where I(*wine*) is the set of everything that can be said to be wine

Problems:

- What about definite descriptions?

The gold on the table weighs fifty grams (Bunt 1985).

It would not do to give the sum of all weights.
So we must impose restrictions on the elements of the set I(*the gold on the table*).

- Identity over time:

The clay that made up those five bowls is identical with the clay that now makes up these two statues.

Which set could ground this identity?
What about the set of all minimal parts of gold?

- Minimal parts:

Modern physics says *gold* and *water* have minimal parts, but semantics? With many mass nouns, like *garbage*, it is not clear what the minimal parts would be. Moreover, mass nouns like *time* and *space* do not seem to have minimal parts. So the semantics of mass nouns should not force them to have minimal parts.

3. Problems for approaches in terms of mereological sums

Link 1983, Gillon 1992.

They focus first on definite descriptions: *the M that Qs* refers to the mereological sum of everything that is some M that Qs.
This takes care of the sentences about the gold or the clay.

Problems:

- But what about *This is wine*?

Proposal: *This is wine* is true iff I(*this*) ≤ I(*wine*)
where ≤ is the parthood relation, and I(*wine*) is the mereological sum of everything that is wine
However, a leg of a chair is not furniture, though it is part of a chair, and a chair is some furniture.

- Moreover, sentences containing mass nouns are liable to collective, distributive, and intermediate construals (cf. Gillon).

This silverware costs a hundred euros

a) The speaker talks of several pieces of silverware considered together: collective construal.

b) The speaker talks of each of several pieces of silverware: distributive construal.

c) The speaker talks of each of several sets of silverware: intermediate construal.

To capture these construals, a notion of covering is needed.
This requires the apparatus of sets, or something as expressive.
Gillon's own approach is thus mixed.

- Consider *The livestock met on the hill*, from Orwell's *Animal farm.*

It is true iff the mereological sum of the livestock met on the hill.

But the right-hand side is in fact very odd: the English predicate *meet* does not seem to apply to mereological sums.

- Finally, suppose some elm is used to make furniture of different styles.

The furniture is heterogeneous is true.
The wood is heterogeneous is false.

But the mereological sum of the wood is identical with the sum of the furniture, so both sentences should have the same truth-value.

=> We will explore a different approach, based on the idea that a mass noun may refer to several things at once. To develop it, we will use a new framework, plural logic.

4. Plural logic

Rayo (2002), Yi (2005, 2006) and McKay (2006).
Plural quantification and predication are recognized as primitive.

Singular term: ‘x’: interpreted as one thing.
Plural term: ‘ys’: interpreted as one or more things, as “a plurality”.
Perplural term: ‘zss’: interpreted as “several pluralities”.

Singular existential quantifier (‘∃x’): there is one thing that…
Plural existential quantifier (‘∃ys’): there are some things that…

A formula like ‘Pcs’ is true when the individuals that interpret the plural constant ‘cs’ collectively satisfy the predicate ‘P’.

x ∠ ys	x is among the ys
ys ∠’ zss	the plural term ‘ys’ denotes “a plurality” among the “several pluralities” denoted by the perplural term ‘zss’

5. A new semantics for mass nouns

- What are the constraints on what counts as M?

a) A thing that counts as M must be self-connected and “maximal”.
b) A thing that counts as M must be self-connected.
c) What counts as M depends on various factors: self-connectedness, function, causality, context, etc.
d) It is “mereological atoms” arranged in a certain way that count as M.

We chose option b), notably in light of sentences like:
What is in this strangely shaped container is wine. In particular, what is in the lower half of this container is also wine.

To ensure this, we adopt two axioms:

$\forall xs\ (Mxs \leftrightarrow \forall y\ (y \sqsubset xs \leftrightarrow My))$ *Distributed reference*

$\forall x\ (Mx \rightarrow \text{self-connected}(x))$ *Self-connectedness*

- What does a definite mass noun phrase refer to?

(Primitive) $z \leq x$ *z is part of x*

(Def) $Oxy \equiv_{def} \exists z\,(z \leq x \wedge z \leq y)$ *x overlaps y*

$ys = \max[zs / Qzs] \equiv_{def}$

$\forall zs\ \forall u\,((Qzs \wedge u \angle zs) \rightarrow \exists v\,(v \angle ys \wedge u \leq v))$

$\wedge\ \forall v\,(v \angle ys \rightarrow \exists zs\,(v \angle zs \wedge Qzs))$

$\wedge\ \neg(\exists u\ \exists v\,(u \angle ys \wedge v \angle ys \wedge u \neq v \wedge Ouv))$

Among all the zs such that Qzs, the ys are the maximal elements for the relation of parthood.

Axiom of maximal reference:

$\exists zs\,(Mzs \wedge Qzs) \rightarrow \exists ys\,(ys = \max[zs / Mzs \wedge Qzs] \wedge Mys)$

Guarantees for instance that *the gold on the table* refers to the three solid bits of gold that are on the table.

- Covering

(Def) x is among° the zss:
$x \angle^{\circ} zss \equiv_{def} \exists ys\ (x \angle ys \wedge ys \angle' zss)$

Covering for a mass noun M
The css are an M-covering of the as just in case:
i) Any thing among° the css is M
$\forall y\ (y \angle^{\circ} css \rightarrow My)$
ii) For anything v, v overlaps some thing among° the css just in case v overlaps one of the as
$\forall v\ (\exists y\ (y \angle^{\circ} css \wedge Ovy) \leftrightarrow \exists w\ (w \angle as \wedge Ovw))$

This is M

The denotation of a mass noun *M* is identified by a plural term, ‘ds’: some thing is M just in case it is one of the ds. The demonstrative *This* refers to one of more things, the as. The sentence is true just in case any of the as is among the ds:

$\forall x\ (x \angle \text{as} \rightarrow x \angle \text{ds})$

This M Ps.

Let the as be the denotation of *this M* (the M demonstrated by the speaker). The interpretation of the sentence depends on the choice of an M-covering of the as. Let the css be the chosen covering. The sentence is true just in case:

$\forall \text{ys}\ (\text{ys} \angle' \text{css} \rightarrow \text{Pys})$

Ex: *This silverware costs a hundred euros*.

The M that Qs Ps.

Let the as be the denotation of *the M that Qs*. They satisfy: as = max[zs / Mzs $\wedge$ Qzs]. The interpretation of the sentence then depends on the choice of an M-covering of the as. Let the css be the chosen covering. The sentence is true just in case:

$\forall$ys (ys $\angle$' css $\rightarrow$ Pys)

Ex: *The gold on the table weighs seven ounces*.

The denotation of the subject is some bits of gold, the as. Each of them is some gold on the table, and is maximal for the relation of parthood.

The collective construal of the sentence is of course the most salient. The covering is then a single "plurality", the as themselves, and the sentence says that the as weigh seven ounces together.

Identity over time:

The clay that made up those five bowls is identical with the clay that now makes up these two statues.

The sentence is made true by a suitable choice of covering, the css, each of which is some clay that has retained its identity over time.

(This does not require the existence of minimal parts.)

At a previous time, the css together made up five bowls.

They have been rearranged, shuffled, to make up two statues.

- What about the furniture and the wood?

Important remark: if the furniture is cut into pieces, it is destroyed, but the wood remains. So a semantics of mass nouns should not identify furniture and wood.

For link and Gillon, the parthood relation is extensional, and this forces the identification of the two.

We require the parthood relation to be a partial ordering, but to satisfy the axiom of strong complementation (cf. Simons 1987) only relative to any given mass noun *M*:

$\forall u\ \forall v\ (\ (Mu \wedge Mv \wedge \neg\ (u \leq v)) \rightarrow \exists x\ (Mx \wedge x \leq u \wedge \neg Oxv)\ )$

In this way, our semantics can coherently deny that furniture and wood are identical.

6. Conclusion

A new semantics of mass nouns cashed out in plural logic.

Avoids problems faced by approaches in terms of sets or mereological sums.

Pays justice to the intuition that a mass noun may refer to several things at once.

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